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Energy Momentum of Marder Universe in Teleparallel Gravity

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Abstract In order to evaluate the energy distribution (due to matter and fields including gravitation) associated with a space-time model of cylindrically-symmetric Marder universe, we consider the Møller, Einstein, Bergmann–Thomson and Landau–Lifshitz energy and momentum definitions in the teleparallel gravity (TG). The energy-momentum distributions are found to be zero. These results are the same as a previous works of Aygün et al., they investigated the same problem in general relativity (GR) by using the Einstein, Møller, Bergmann–Thomson, Landau–Lifshitz (LL), Papapetrou, Qadir–Sharif and Weinberg's definitions. These results support the viewpoints of Banerjee–Sen, Xulu, Radinschi and Aydoğdu–Saltı. Another point is that our study agree with previous works of Cooperstock–Israelit, Rosen, Johri et al. This paper indicates an important point that these energy-momentum definitions agree with each other not only in general relativity but also in teleparallel gravity. It is also independent of the teleparallel dimensionless coupling constants, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model.

Keywords Marder universe · Energy-momentum distributions · Møller prescription · Teleparallel gravity

1 Introduction

From the advent of the general theory of relativity, various methods have been proposed to deduce the conservation laws that characterize the gravitational systems. Nevertheless,

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recently this problem also argued in an alternative gravitation theory, namely teleparallel gravity. Energy and momentum of the universe play an important role as they provide the first integrals of equations of motions, helping one to solve otherwise intractable problems [32]. Furthermore, the energy content in a sphere of radius R in a given space-time gives a taste of the effective gravitational mass that a test particle situated at the same distance from the gravitating object experiences. A large number of researchers have devoted considerable attention to the problem of finding the energy as well as momentum and angular momentum associated with various space-times. The first of such attempts was made by Einstein who proposed an expression for the energy-momentum distribution of the gravitational field. There have been many attempts to resolve the energy-momentum problem [11, 25, 29, 30, 33, 35, 64, 71, 76]. There exists an opinion that the energy and momentum definitions are not useful to get finite and meaningful results in a given geometry. These complexes give the meaningful results when we transform the line-element into the quasi-Cartesian coordinates. The energy and momentum complex of Møller gives the possibility to make the calculations in any coordinate system [36]. Rosen [50] calculated the total energy of a FRW metric and found it to be zero, using Einstein's energy-momentum definitions. The total energy of the same universe was obtained by Johri et al. [23] with the Landau–Lifshitz energy-momentum complex. They found that it is zero at all times. Moreover, they showed that the total energy enclosed within any finite volume of a spatially flat FRW universe is vanishing. Banerjee and Sen [10] who considered Bianchi type-I space-times, showed that the energy-momentum density is zero everywhere, with the energy-momentum definition of Einstein. Virbhadra and his collaborators have considered many space-time models and showed that several energy-momentum complexes give the same and acceptable results for a given space-time model [1, 13–16, 36–50, 66–70, 72–75], Virbhadra [73], using the general relativity versions of energy and momentum complexes of Einstein, Landau–Lifshitz, Papapetrou and Weinberg's for a general non-static spherically symmetric metric of the Kerr–Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in Penrose energy-momentum complex. Sharif et al. [58–63] have investigated in detail energy-momentum problem in the theory of general relativity and teleparallel gravity by using different space times and different definitions. Albrow [2] and Tryon [65] suggested that in our universe, all conserved quantities have to vanish. Tryon's big bang model predicted a homogeneous, isotropic and closed universe including of matter and anti-matter equally. They argue that any closed universe has zero energy. The subject of the energy-momentum distributions of the closed universes was opened by an interesting work of Cooperstock and Israelit [17]. They found the zero value energy for a closed homogeneous isotropic universe described by a Friedmann–Robertson–Walker (FRW) metric in the context of general relativity. And in teleparallel gravity, there have been some attempts to show the teleparallel gravitational energy-momentum definitions give the same results as obtained by using the general relativistic ones [3–7, 52–57]. In his recent paper, Vargas [71] using the Einstein and Landau–Lifshitz complexes, calculated the energy-momentum density of the Friedman-Robertson-Walker space-time and showed that the result is the same as obtained in general relativity. Aygün [8] have investigated the energy-momentum problem of Marder universe in general relativity by using the Einstein, Møller, Bergmann–Thomson, Landau–Lifshitz (LL), Papapetrou, Qadir-Sharif and Weinberg's definitions and found same results.

The basic purpose of this paper is to obtain the total energy and momentum of Marder universe by using the energy-momentum expressions of Møller, Einstein, Bergmann–Thomson and Landau–Lifshitz in teleparallel theory of gravity. We will proceed according to the following scheme. In Sect. 2, we briefly present cylindrically-symmetric Marder type space-time and carry out some necessary calculations for this model. In Sect. 3 we will give the Einstein, Bergmann–Thomson and Landau–Lifshitz energy momentum complexes in teleparallel gravity and then we will compute the energy-momentum densities. In Sect. 4, we will give the Møller energy-momentum complex and then we will get the Møller energy-momentum density in teleparallel gravity. Finally, we summarize and discuss our results. Throughout this paper, the Latin indices $(i, j, \ldots)$ represent the vector number and the Greek $(\mu, \nu, \ldots)$ represent the vector components; all indices run from 0 to 3. We use geometrized units where $G = 1$ and $c = 1$.

2 The Cylindrically-Symmetric Marder Type Space-Time

In this paper we consider Marder's cylindrically-symmetric metric in the form [27]

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B, C are functions of t only. The model becomes conformal to flat space time in particular cases. This metric represents the anisotropic homogeneous universe. Cosmological models which are anisotropic and homogeneous have a significant role in the description of the universe in the early stages of its evolution [9]. Also this metric is Cylindrically-symmetric. The cylindrically-symmetric space-time plays an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored [34]. Cylindrically-symmetric cosmological models have made a significant contribution in under standing some essential features of the universe such as the formation of galaxies during the early stages of their evolution [24]. The cylindrically-symmetric space-times representing material distribution were obtained by Marder [27]. The metric can be transformed to the Bianchi type-I form by the coordinate transformation $t \to \int A(t)dt$. The original work of Bianchi [12] has been reorganized into a contemporary formalism by theoretical cosmologists. The Bianchi type space-times generally defined by the following metric

$$ds^2 = -dt^2 + dl^2 \quad \text{where } dl^2 = g_{ab}dx^a dx^b$$

where dl^2 is the 3-dimensional line element and Latin indices take value from 1 to 3. The Bianchi cosmologies play an important role in theoretical cosmology and have been much studied since the 1960s. Such cosmologies provide interesting generalizations of the standard Friedmann–Lemaître models of cosmology (which are based on the spatially homogeneous and isotropic Robertson–Walker geometries, with spatial sections of constant curvature). The Bianchi universe models are spatially homogeneous cosmological models that in general are anisotropic. The Bianchi models are defined to be the family of cosmologies in which there is a 3-dimensional group of isometries G_3 acting on spacelike 3-surfaces; making these surfaces of homogeneity in space-time (all physical quantities are necessarily constant on them). These models can be used to analyze aspects of the physical Universe which pertain to or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe, and the question of the isotropization of the universe itself [26]. Spatially homogeneous cosmologies also play an important role in attempts to understand the structure and properties of the space of all cosmological solutions of Einstein field equations. For the line element (1), $g_{\mu\nu}$ is defined by

$$(g_{\mu\nu}) = \begin{pmatrix} -A^2 & 0 & 0 & 0 \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & B^2 & 0 \\ 0 & 0 & 0 & C^2 \end{pmatrix}. \tag{2}$$

The inverse of $g_{\mu\nu}$ is evidently,

$$(g^{\mu\nu}) = \begin{pmatrix} -A^{-2} & 0 & 0 & 0 \\ 0 & A^{-2} & 0 & 0 \\ 0 & 0 & B^{-2} & 0 \\ 0 & 0 & 0 & C^{-2} \end{pmatrix}. \tag{3}$$

Introducing the tetrad ω^i $(i = 0, 1, 2, 3)$ are given by

$$\omega^0 = Adt, \qquad \omega^1 = Adx, \qquad \omega^2 = Bdy, \qquad \omega^3 = Cdz \tag{4}$$

the metric (1) can be expressed in the simple form

$$ds^2 = -(\omega^0)^2 + (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2. \tag{5}$$

The non-trivial tetrad field induces a teleparallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu. \tag{6}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \text{diag}(A, A, B, C). \tag{7}$$

$$h_a{}^\mu = \text{diag}\left(\frac{1}{A}, \frac{1}{A}, \frac{1}{B}, \frac{1}{C}\right). \tag{8}$$

3 Einstein, Bergmann–Thomson and Landau–Lifshitz Energy Momentum Prescriptions in Teleparallel Gravity

The teleparallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [77]. In the theory of the teleparallel gravity, gravitation is attributed to torsion [19] which plays the role of a force [18] and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting place of teleparallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent when considered from the teleparallel point of view.

The Einstein, Bergmann–Thomson and Landau–Lifshitz's energy-momentum complexes in teleparallel gravity [71] are given as follows:

$$hE^\mu{}_\nu = \frac{1}{4\pi}\partial_\lambda(U_\nu{}^{\mu\lambda}), \tag{9}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(g^{\mu\beta}U_\beta{}^{\nu\lambda}), \tag{10}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda (hg^{\mu\beta}U_\beta{}^{\nu\lambda}) \tag{11}$$

where $U_\beta{}^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_\beta{}^{\nu\lambda} = hS_\beta{}^{\nu\lambda} \tag{12}$$

where $h = \det(h^a{}_\mu)$ and $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = e_1 T^{\mu\nu\lambda} + \frac{e_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{e_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_\beta - g^{\nu\mu}T^{\beta\lambda}{}_\beta) \tag{13}$$

with e_1, e_2 and e_3 the three dimensionless coupling constants of teleparallel gravity [20]. For the teleparallel equivalent of general relativity the specific choice of these three constants are:

$$e_1 = \frac{1}{4}, \qquad e_2 = \frac{1}{2}, \qquad e_3 = -1. \tag{14}$$

To calculate this tensor, firstly we must calculate Weitzenböck connection:

$$\Gamma^\alpha{}_{\mu\nu} = h_a{}^\alpha \partial_\nu h^a{}_\mu \tag{15}$$

and torsion of the Weitzenböck connection:

$$T^\mu{}_{\nu\lambda} = \Gamma^\mu{}_{\lambda\nu} - \Gamma^\mu{}_{\nu\lambda}. \tag{16}$$

The energy and momentum distributions in the complexes of Einstein, Bergmann–Thomson and Landau–Lifshitz in the teleparallel gravity are given by the following equations, respectively,

$$P_\mu^E = \int_\Sigma hE^0{}_\mu dxdydz, \tag{17}$$

$$P_\mu^B = \int_\Sigma hB^0{}_\mu dxdydz, \tag{18}$$

$$P_\mu^L = \int_\Sigma hL^0{}_\mu dxdydz. \tag{19}$$

P_μ is called the momentum four-vector, P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t = \text{constant}$.

From (15), the non-vanishing Weitzenböck connection components are obtained as

$$\begin{aligned}
\Gamma^1_{10} &= \frac{A_t}{A}, \\
\Gamma^2_{20} &= \frac{B_t}{B}, \\
\Gamma^3_{30} &= \frac{C_t}{C}, \\
\Gamma^0_{00} &= \frac{A_t}{A},
\end{aligned} \tag{20}$$

where t indices describe the derivative with respect to t. The corresponding non-vanishing torsion components are obtained as

$$\begin{aligned} T^1_{\ 10} &= -T^1_{\ 01} = -\frac{A_t}{A}, \\ T^2_{\ 20} &= -T^2_{\ 02} = -\frac{B_t}{B}, \\ T^3_{\ 30} &= -T^3_{\ 03} = \frac{C_t}{C}. \end{aligned} \tag{21}$$

Using these results with (13), the non-vanishing components of the tensor $S^{\mu\nu\lambda}$ are obtained as

$$\begin{aligned} S^{110} &= -S^{101} = -\frac{(BC)_t}{2A^4BC}, \\ S^{220} &= -S^{202} = -\frac{(AC)_t}{2A^3B^2C}, \\ S^{330} &= -S^{303} = -\frac{(AB)_t}{2A^3C^2B}. \end{aligned} \tag{22}$$

From (12), the required components of Freud's super potential are calculated as

$$\begin{aligned} U_1{}^{10} &= -U_1{}^{01} = -\frac{(BC)_t}{2}, \\ U_2{}^{20} &= -U_2{}^{02} = -\frac{B(AC)_t}{2A}, \\ U_3{}^{30} &= -U_3{}^{03} = -\frac{C(AB)_t}{2A}. \end{aligned} \tag{23}$$

Substituting (23) into (9), (10) and (11), we get Einstein, Bergmann–Thomson and Landau–Lifshitz energy-momentum densities in teleparallel gravity, respectively.

$$hE^{\mu}{}_{\nu} = hB^{\mu\nu} = hL^{\mu\nu} = 0. \tag{24}$$

Substituting these results into (17), (18) and (19) we get total energy-momentum distributions of the Marder universe for Einstein, Bergmann–Thomson and Landau–Lifshitz prescriptions in teleparallel gravity, respectively.

$$P^E_{\mu} = 0, \tag{25}$$

$$P^B_{\mu} = 0, \tag{26}$$

$$P^L_{\mu} = 0. \tag{27}$$

4 Møller Energy-Momentum Prescription in Teleparallel Gravity

Møller modified general relativity by constructing a new field theory in teleparallel space [31]. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian Space [31]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [51] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [28] showed that Møller theory is a special case of Poincare gauge theory [20–22].

The super-potential of Møller in teleparallel gravity is given by Mikhail et al. [29] as

$$\Im_{\mu}^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} \Upsilon^{\tau\nu\beta}_{\chi\rho\sigma} [\Phi^{\rho} g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \gamma^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu} \gamma^{\sigma\rho\chi}] \tag{28}$$

where $\Upsilon^{\tau\nu\beta}_{\chi\rho\sigma}$ is

$$\Upsilon^{\tau\nu\beta}_{\chi\rho\sigma} = \delta^{\tau}_{\chi} g^{\nu\beta}_{\rho\sigma} + \delta^{\tau}_{\rho} g^{\nu\beta}_{\sigma\chi} - \delta^{\tau}_{\sigma} g^{\nu\beta}_{\chi\rho} \tag{29}$$

with $g^{\nu\beta}_{\rho\sigma}$ being a tensor defined by

$$g^{\nu\beta}_{\rho\sigma} = \delta^{\nu}_{\rho} \delta^{\beta}_{\sigma} - \delta^{\nu}_{\sigma} \delta^{\beta}_{\rho} \tag{30}$$

and $\gamma_{\mu\nu\beta}$ is the con-torsion tensor given by

$$\gamma_{\mu\nu\beta} = h_{i\mu} h^{i}_{\nu;\beta} \tag{31}$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbols. g is the determinant of the $g_{\mu\nu}$, Φ_{μ} is the basic vector field defined by

$$\Phi_{\mu} = \gamma^{\rho}_{\mu\rho} \tag{32}$$

κ is the Einstein constant and λ is the free dimension-less parameter. The energy may be expressed by the surface integral

$$E^{\text{Moller}}_{TG} = \lim_{r\to\infty} \int_{r=\text{constant}} \Im_{0}^{0\alpha} n_{\alpha} dS \tag{33}$$

where n_{α} is the unit three-vector normal to the surface element dS. Taking the results which are given by (7) and (8) into equation (31) we get the non-vanishing components of $\gamma_{\mu\nu\beta}$ as:

$$\begin{aligned} \gamma_{011} &= -\gamma_{101} = AA_t, \\ \gamma_{022} &= -\gamma_{202} = BB_t, \\ \gamma_{033} &= -\gamma_{303} = CC_t. \end{aligned} \tag{34}$$

Using this results, we find following non-vanishing component of basic vector field:

$$\Phi^{0} = \frac{1}{A^2}\left(\frac{A_t}{A} + \frac{B_t}{B} + \frac{C_t}{C}\right). \tag{35}$$

From (28) with the results which are given in (34) and (35) we find the required components of Møller's super-potential are vanishing. Substituting this results into (33) we get

$$E^{\text{Moller}}_{TG} = 0. \tag{36}$$

5 Summary and Discussion

The subject of energy-momentum localization has generated great deal of interest in both general relativity and teleparallel gravity, although it has been the focus of some debate. In this paper, we used the cylindrically-symmetric Marder type space-time and calculated the energy-momentum densities of this space-time model with the Bergmann–Thomson, Einstein, Landau–Lifshitz and Møller energy-momentum definitions in teleparallel gravity. We found that; the total energy and momentum components have zero net value all investigated energy momentum densities, because the energy contributions from the matter and gravitational field inside an arbitrary two-surface in the case of anisotropic model of the universe based on cylindrically-symmetric Marder metric, cancel each other. Therefore the total energy and momentum components have zero value and different energy-momentum complexes give same results in Marder space-time. This results are the same as a previous works of Aygün et al. [8], they investigated the same problem in general relativity by using the Einstein, Møller, Bergmann–Thomson, Landau–Lifshitz (LL), Papapetrou, Qadir-Sharif and Weinberg's definitions. We found that these two gravitational theories give the same results for the total energy (E) and momentum (M) in General relativity and Teleparallel Gravity:

$$E_{GR}^{\text{Marder}} = E_{TP}^{\text{Marder}} = 0, \qquad M_{GR}^{\text{Marder}} = M_{TP}^{\text{Marder}} = 0.$$

This paper demonstrates the important point that these energy momentum definitions are identical not only in general relativity but also in teleparallel gravity. It is also independent of the teleparallel dimensionless coupling constants, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model. When we apply coordinate transformation $t \to \int A(t)dt$, we obtain Bianchi type I universe defined by the line element;

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2$$

and the metric given above reduces to some well-known space times in special cases.

(A) For $A = B = C = R(t)$ and transforming the line element to t, x, y, z coordinates according to

$$x = r\sin(\theta)\cos(\phi), \qquad y = r\sin(\theta)\sin(\phi), \qquad z = r\cos(\theta)$$

gives

$$ds^2 = -dt^2 + R(t)^2[dr^2 + r^2(d\theta^2 + \sin(\theta)^2 d\phi^2)].$$

which describes well-known spatially flat Friedmann–Robertson–Walker (FRW) space-time.

(B) For $A = e^{l(t)}$, $B = e^{m(t)}$ and $C = e^{n(t)}$, then the line element describes the well-known Bianchi-I type metric.

(C) For $A = t^a$, $B = t^b$ and $C = t^c$, (where a, b, c are constants), then we obtain the well-known Kasner-type model.

The results of energy and momentum distributions of these metrics are same as our results,

$$\begin{aligned} &E_{\text{FRW}} = M_{\text{FRW}} = 0, \\ &E_{\text{Bianchi}I} = M_{\text{Bianchi}I} = 0, \\ &E_{\text{Kasner}} = M_{\text{Kasner}} = 0. \end{aligned} \tag{37}$$

Finally, the results that the total energy-momentum densities are vanishing everywhere. These results supports the viewpoints of Banerjee–Sen, Xulu, Radinschi and Aydoğdu–Saltı. Another point is that our study agree with previous works of Tryon, Cooperstock–Israelit, Rosen, Johri et al.

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References

1. Aguirregabiria, J.M., Chamorro, A., Virbhadra, K.S.: Gen. Relativ. Gravit. **28**, 1393 (1996)
2. Albrow, M.G.: Nature **241**, 56 (1973)
3. Aydogdu, O.: Int. J. Mod. Phys. A **21**, 3845 (2006)
4. Aydogdu, O.: Int. J. Mod. Phys. D **15**, 459 (2006)
5. Aydogdu, O.: Fortschr. Phys. **54**, 246 (2006)
6. Aydogdu, O., Saltı, M., Korunur, M.: Acta Phys. Slov. **55**, 537 (2005)
7. Havare, A., Korunur, M., Saltı, M.: Astrophys. Space Sci. **301**, 43 (2006)
8. Aygün, S., Aygün, M., Tarhan, İ.: Pramana J. Phys. **68**(1), 21 (2007), gr-qc/0607102
9. Bali, R., Jain, D.R.: Int. J. Theor. Phys. **28**, 8 (1989)
10. Banerjee, N., Sen, S.: Pramana J. Phys. **49**, 609 (1997)
11. Bergmann, P.G., Thomson, R.: Phys. Rev. **89**, 400 (1953)
12. Bianchi, L.: Mem. Mat. Fis. Soc. Ital. Sci. **11**, 267 (1898)
13. Chamorro, A., Virbhadra, K.S.: Pramana J. Phys. **45**, 181 (1995)
14. Chamorro, A., Virbhadra, K.S.: Int. Mod. Phys. D **5**, 251 (1997)
15. Cooperstock, F.I., Richardson, S.A. In: Proc. 4th Canadiand Conf. on General Relativity and Relativistic Astrophysics. World Scientific, Singapore (1991)
16. Cooperstock, F.I.: Gen. Rel. Gravit. **26**, 323 (1994)
17. Cooperstock, F.I., Israelit, M.: Found. Phys. **25**, 631 (1995)
18. de Andrade, V.V., Pereira, J.G.: Phys. Rev. D **56**, 4689 (1997)
19. Hayashi, K., Shirafuji, T.: Phys. Rev. D **19**, 3524 (1978)
20. Hayashi, K., Shirafuji, T.: Prog. Theor. Phys. **64**, 866 (1980)
21. Hayashi, K., Shirafuji, T.: Prog. Theor. Phys. **65**, 525 (1980)
22. Hehl, F.W., Nitsch, J., von der Heyde, P. In: Held, A. (ed.), General Relativity and Gravitation. Plenum, New York (1980)
23. Johri, J.B., Kalligas, D., Singh, G.P., Everitt, C.W.F.: Gen. Relativ. Gravit. **27**, 323 (1995)
24. Kılınç, C.B.: Astrophys. Space Sci. **222**, 171 (1994)
25. Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields. 4th edn. Pergamon, Oxford (1987)
26. MacCallum, M.A.H.: Einstein Centenary Survey. Cambridge University Press, Cambridge (1979)
27. Marder, L.: Proc. Roy. Soc. Lond. A **246**, 133 (1958)
28. Meyer, H.: Gen. Relativ. Gravit. **14**, 531 (1982)
29. Mikhail, F.I., Wanas, M.I., Hindawi, A., Lashin, E.I.: Int. J. Theor. Phys. **32**, 1627 (1993)
30. Møller, C.: Ann. Phys. (N.Y.) **12**, 118 (1961)
31. Møller, C.: Mat. Fys. Medd. K. Vidensk. Selsk. **1**, 10 (1961)
32. Namad-Achar, E., Schutz, B.F.: Gen. Relativ. Gravit. **19**, 7 (1987)
33. Papapetrou, A.: Proc. R. Irish. Acad. **A52**, 11 (1948)
34. Prakash, S.: Astrophys. Space Sci. **111**, 383 (1985)
35. Qadir, A., Sharif, M.: Phys. Lett. A **167**, 331 (1992)
36. Radinschi, I.: Mod. Phys. Lett. A **15**, 2171 (2000)
37. Radinschi, I.: Fiz. B **9**, 43 (2000)
38. Radinschi, I.: Fiz. B **14**, 3 (2005)
39. Radinschi, I.: Phys. Ser. **42**, 11 (2001)
40. Radinschi, I.: Mod. Phys. Lett. A **16**, 673 (2001)
41. Radinschi, I.: Acta Phys. Hung. **12**, 47 (2000)
42. Radinschi, I.: Acta Phys. Slov. **50**, 609 (2000)
43. Radinschi, I.: Chin. J. Phys. **39**, 231 (2001)
44. Radinschi, I.: Chin. J. Phys. **39**, 393 (2001)
45. Yang, I.-C., Radinschi, I.: Chin. J. Phys. **41**, 326 (2003)

46. Yang, I.-C., Radinschi, I.: Chin. J. Phys. **42**, 40 (2004)
47. Yang, I.-C., Radinschi, I.: Mod. Phys. Lett. A **17**, 1159 (2002)
48. Radinschi, I., Grammenos, Th.: Int. J. Mod. Phys. A **21**, 2853 (2006)
49. Rosen, N., Virbhadra, K.S.: Gen. Relativ. Gravit. **25**, 429 (1993)
50. Rosen, N.: Gen. Relativ. Gravit. **26**, 323 (1995)
51. Saez, D.: Phys. Rev. D **27**, 2839 (1983)
52. Saltı, M., Havare, A.: Int. J. Mod. Phys. A **20**, 2169 (2005)
53. Saltı, M.: Astrophys. Space Sci. **299**, 159 (2005)
54. Saltı, M.: Mod. Phys. Lett. A **20**, 2175 (2005)
55. Saltı, M.: Acta Phys. Slov. **55**, 563 (2005)
56. Saltı, M.: Czech. J. Phys. **56**, 177 (2006)
57. Saltı, M., Aydogdu, O.: Found. Phys. Lett. **19**, 269 (2006)
58. Sharif, M.: Int. J. Mod. Phys. A **17**, 1175 (2002)
59. Sharif, M.: Int. J. Mod. Phys. D **13**, 1019 (2004)
60. Sharif, M.: Nuovo Cimento B **19**, 463 (2004)
61. Sharif, M.: Nuovo Cimento B **118**, 669 (2003)
62. Sharif, M., Amir, M.J.: Gen. Relativ. Gravit. **38**, 1735 (2006)
63. Sharif, M., Fatima, T.: Astrophys. Space Sci. **302**, 217 (2006)
64. Tolman, R.C.: Relativity, Thermodinamics and Cosmology, p. 227. Oxford University Press, London (1934)
65. Tryon, E.P.: Nature **246**, 396 (1973)
66. Vagenas, E.C.: Int. J. Mod. Phys. A **18**, 5781 (2003)
67. Vagenas, E.C.: Int. J. Mod. Phys. A **18**, 5949 (2003)
68. Vagenas, E.C.: Mod. Phys. Lett. A **19**, 213 (2004)
69. Vagenas, E.C.: Int. J. Mod. Phys. D **14**, 573 (2005)
70. Vagenas, E.C.: Mod. Phys. Lett. A **21**, 1974 (2006)
71. Vargas, T.: Gen. Relativ. Gravit. **36**, 1255 (2004)
72. Virbhadra, K.S.: Phys. Rev. D **41**, 1086 (1990)
73. Virbhadra, K.S.: Phys. Rev. D **60**, 104041 (1999)
74. Virbhadra, K.S.: Pramana J. Phys. **45**, 215 (1995)
75. Virbhadra, K.S., Parikh, J.C.: Phys. Lett. B **331**, 302 (1994)
76. Weinberg, S.: Gravitation and Cosmology: Principle and Applications of General Theory of Relativity. Wiley, New York (1972)
77. Weitzenböck, R.: Invarianttent Theorie. Noordhoff, Gronningen (1923)